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mist: Multispectral Image Similarity Transformation

Contract Number: NAS5-32357 Final Report



Introduction

This document describes work intended to aid in the problem of automatically registering two images of dissimilar type. This work was performed under NASA contract number NAS5-32357. It was performed by Sridhar Srinivasan, Carl Stevens, Les Elkins, Radha Poovendran, and Srinivasan Raghavan,.

Registration Algorithms as Implemented in mist Code

The algorithms implemented in this software attempt to determine a transformation from one image to another. The registration software can be used to register similar or dissimilar multi-sensor imagery as long as the transformation between this imagery is restricted to rotation, translation, and scaling (collectively known as a "similarity transformation"). This composite transformation can be expressed as:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = s \begin{bmatrix} T_x \\ T_y \end{bmatrix} + s \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

where:

(x',y') is the transformed point in the second image, image2, corresponding to (x,y) in the first image, image1,

s is the common scale by which image1 was expanded to create image2,

 T_{x} , T_{y} are the units that image 1 was translated in the x and y axes,

 θ is the angle by which image 1 was rotated, and

(x,y) is the source point in the original image.

The algorithms which determine the scale, translation, and rotation of the two images are based on lines extracted from the original images. By working with these lines rather than the images themselves, the algorithm can deal with differing types of imagery (visible, IR, AVHRR, etc.).

The algorithm performs five steps:

- Extract edges from image files,
- Extract line segments from edges,
- Determine rotation parameter based on lines segments,
- Determine scale parameter based on line segments, and
- Determine translation parameter based on line segments.

Each of these will be dealt with in greater detail below.

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Edge Extraction

The quality of edges obtained using any given edge detection technique is very critical to the output of the line extraction algorithm. The desirable features in an edge detection algorithm therefore include:

- High detection rates: The probability of missing a valid edge should be low. The probability of falsely identifying a nonexistent edge should be low as well. The probability of false alarms, as well as the probability of a missed edge, are both monotonically decreasing functions in terms of the signal-to-noise ratio. If the noise is assumed to be Gaussian, the probability of a false alarm is given by the complementary error function, and the probability of detection is the same function shifted by the square root of the signal to noise ratio. Hence the property of high detection is equivalent to maximizing the signal to noise ratio for a given detector.
- Localization of edges: The estimated edge points should be as close as possible to the true
 edge points.
- Uniqueness of the edge: The solution to the coordinates of the edge point should be unique at every point.

Canny¹ showed that the solution to these three constraints leads to a linear match filter. From numerical optimizations, Canny also noted that an efficient approximation to the edge function is the derivative of the Gaussian,

$$G(x) = \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

where x is the location of the edge and σ is the standard deviation of the Gaussian function.

To produce the list of valid edges, the code does the following:

- Computes the separable Gaussian functions in the x and y directions.
- Computes the image gradient.
- Convolves the image gradient with the gradient of the Gaussian.
- Generates the edge map by starting at points above a "high" threshold, and connecting to the points that are above the "low" threshold (in this implementation, "high" is 255 fully activated, and "low" is one or greater).

Line Segment Extraction

Once the edges are extracted, they must be converted into lines. This is done by grouping valid edge points that lie along lines and creating line segments. Valid lines are determined by edge density and line length.

¹ Canny, J., "A Computational Approach to Edge Detection," IEEE Transactions on PAMI, Vol. 8, No. 6, November 1986, pp. 679-698

This method generates many valid line segments, but will also generate a number of spurious ones. Inclusion of these segments in the parameter estimation algorithms will skew the estimated transformation parameter values. It is therefore necessary to use thresholding operations which will eliminate as many spurious line segments as possible. The current version of the code picks rather permissive parameters, which nonetheless keeps the algorithm from generating many of the very small lines.

The algorithms used in the following sections contain code that will increase in computational cost greatly as the number of lines used increases. Thus once the lines are generated, it is still desirable to further reduce their number. This is done by choosing a minimum length threshold based on an examination of the histogram. The threshold chosen by the code as it stands now is the value one quartile from the lowest value, thus three quarters of the lines remain after this additional restriction.

Determination of the Rotation Parameter

Given a set of lines from each image of the pair, we wish to determine the rotation angle needed to rotate the lines in one image to those in the second. This requirement is complicated by the fact that there may not be a high degree of correspondence between the two sets of lines, and by the possibility that the two sets of lines are at different scales and may be shifted spatially. It is therefore desirable to use techniques based on histogram solutions, which will give us the maximum likelihood solution for the parameter estimation problems at hand.

The first step in computing the rotation is to determine the direction of each line. The direction is given as the angle with respect to the x axis, and is constrained to be between $\left(\frac{\pi}{2}\right)$ and $\left(-\frac{\pi}{2}\right)$. Then, for each line

in the first image, the difference in angle to each line in the second image is computed. This then gives us a list of the difference in direction between all lines in the first file to all lines in the second file. While in practice the images we have used have contained a manageable number of lines, note that the number of values stored (number of lines above threshold length in image one times number of lines above threshold in image two) can get very large, and it may be desirable to increase the length threshold for some images.

To analyze this list, we perform a histogram analysis of the difference values. Since in the problem domain we are examining implies that we have at least coarse correspondence between the image pair, we typically assume that the rotational angle between the two images is between -10 degrees and +10 degrees. This default can be overridden to look for rotational correlation over larger angles. We can then build our histogram accordingly, ignoring angle differences outside these bounds.

The optimum number of bins in the histogram is a function of the desired accuracy (which implies a small bin size, and thus a large number of bins) and the amount of data to be examined (with sparse data, bin sizes too small might not be filled adequately, and thus might give spurious results, implying the need for a small number of bins). In this implementation, 100 bins are used, giving a bin size of

$$\frac{10 - (-10)}{100} = 0.2$$

degrees per bin.

The histogram is then filtered by two passes with a (1,2,1) mean filter, and the angle value corresponding to the bin with the maximum number of entries is chosen as the rotational value. The set of lines

corresponding to the second image is then rotated by the negative of this determined value for further processing. This rotation is performed about the center of the image.

Determination of the Scale Parameter

Even though lines are robust enough to use as features, some line segments in each image may be broken or missing altogether due to various reasons (noise, poor contrast, etc.). Hence simple examination of the ratio of scales of corresponding lines (as we examined the angles of corresponding lines above) is insufficient to give good results. The method implemented in this project composes the given lines into triangles, and determines the ratio of the area of the triangles in the first image to the triangles in the second image.

To form the triangles, the list of lines is exhaustively searched. To find a potential triangle from the first image, three lines from that image, 11, 12, and 13, are chosen such that their directions differ from each other by more than some angle tolerance δ . In the current implementation, δ is 25 degrees. This prevents excessively slivered triangles from being considered, as they would have a small area which might skew the results. Note that we are not looking for actual triangles in the line segments, but rather lines which can be extended to form triangles for consideration. To find a corresponding triangle in the second image, we find lines 11', 12', and 13' such that 11', 12', and 13' have directions greater than δ degrees apart, and which additionally fulfill the requirement that each corresponding line in the two triangles is within some tolerance β : that is, the direction of 11 and 11' differ by less than β degrees, 12 and 12' differ by less than β degrees, and 13 and 13' differ by less than β degrees. β defaults to ten degrees in the current implementation. Note that for any triangle in the first image there may be several in the second image that fulfill these requirements.

Once two sets of lines are found, the area of the triangles they represent must be determined. This is computed by a determinant:

$$A = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix} \Rightarrow A = \frac{1}{2} (x_2 y_3 - x_3 y_2 + x_3 y_1 - x_1 y_3 + x_1 y_2 - x_2 y_1)$$

where (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) are the coordinates of the vertices of the triangle. This computation is performed for both triangles. Since the x and y axes in the second image are scaled by s, a triangle from the first image with area A should have a corresponding triangle in the second image with an area roughly equal to s^2A . Thus the square root of the ratio of a triangle's area in the second image to the area of a corresponding triangle in the first image will be the scale factor between these triangles. Note that since this computation depends on the triangle's area only, it is invariant with respect to both translation and rotation. Also, since particularly small areas in the denominator can skew the results, we ignore small triangles.

With this computation performed on the large number of corresponding triangles present in an image, then a large number of ratios will be available in a list. We then perform histogram analysis as we did in the case of rotation to determine the most common scaling factor, except the histograms are built in log space. This factor is then used to invert the scaling in the line list from the second image, in order to obtain a true translation estimation, as described next.

Determination of the Translation Parameter

To find the translation parameters, the algorithm again builds a list of triangles as in the scaling code, this time using the derotated and descaled lines. The centroid of each triangle is generated by:

$$(\bar{x}, \bar{y}) = \left(\frac{1}{3}(x_1 + x_2 + x_3), \frac{1}{3}(y_1 + y_2 + y_3)\right)$$

A list is then generated of all possible translations between the centers of corresponding triangles. Again using histogram methods, we determine the x and y values that occur the maximum number of times in the range of interest. These are the translation values, T_x and T_y .

With all four parameters now known, the code performs the transformation on the second image to rotate, scale, and translate it to correspond to the first.

C Code Function Description

This section describes the function names in the C code and the parameters they take and return. In most cases, the algorithms have been discussed in the previous section.

register_images

register_images makes the calls to the supporting routines to perform the line identification and correlation.

```
void register images (
                              /* primary input file name (pgm file) */
  char
         *in1,
                              /* secondary input file name (pgm file) */
         *in2,
  char
                              /* output file name (pgm file) */
         *out,
  char
                             /* sigma for canny edge detection */
/* minimum length of poly-line segment */
  float sigma,
  float min_segment,
float max distance,
                             /* max. distance of poly line from edge */
                              /* min. line length considered */
  float thresh,
                             /* range of (+/-) permissible rotation */
  float rotation_range,
                             /* range of scales in percent (+/-) 100 */
  float scale_range,
                              /* range of shifts in pixels */
  float shift_range
```

These parameters are primarily passed to the supporting functions. If a zero is passed for any of the numeric parameters, then default values will be chosen. This function also computes the desired minimum line length if it is passed a zero for thresh (it chooses the value one quartile into the sorted list of lengths).

compare

compare defines a comparison function for two pointers to floating point values. This function is used from C's built in qsort function.

```
int compare (const void *a, const void *b)
```

This function is not intended to be directly called by the user.

compute rotation

compute_rotation determines the rotation offset between two sets of lines.

```
float compute_rotation(float *angl, /* Array of angles for lines in first image */
float *ang2, /* " " " " " second " */
int dlength1, /* Number of lines from first image. */
int dlength2, /* " " second " */
float *length1, /* Length of lines from first image. */
float *length2, /* " " " second " */
float thresh, /* Minimum line length considered. */
float range) /* Max number of degrees rot. considered */
```

The number returned is the rotation angle (in degrees).

derotate

derotate performs the derotation.

Note that the rotation is performed with respect to the center of the image (so the endpoints are translated by (-1/2 xsize, -1/2 ysize) before rotation, then translated back by the same amount before storage).

differs

differs compares two floating point numbers (angles in degrees).

The function returns one (true) if they are greater than some number of degrees from each other, and zero (false) if they are not. The difference threshold is defined in the routine as a constant, and is 25 in the current implementation. This function also deals with wraparound. If it compares, for example, line segments with angles 2 degrees and 178 degrees, it will treat them as differing by 4 degrees rather than 176.

is_close

is_close compares two floating point numbers (angles in degrees).

The function returns one (true) if they are within some number of degrees of each other, and zero (false) if they are not. The difference threshold is defined in the routine as a constant, and is 10 in the current implementation. This function also deals with wraparound, so if it compares, for example, line segments with angles 2 degrees and 178 degrees, it will treat them as differing by 4 degrees rather than 176.

compute_param

compute_param takes a list of line endpoints and computes the angle and length of each segment.

```
float *px) /* px contains x sin ang - y cos of ang */
```

This routine computes a number of numeric values for later use, as indicated above.

rescale

rescale will rescale the line endpoints given the data given a scaling factor.

The endpoint values are scaled about the origin.

Intersection

intersection will compute the intersection between the two lines specified.

```
float *intersection (float sin1, /* sine of angle of first line */
float cos1, /* cosine of angle of first line */
float p1, /* intercept representation of first line */
float sin2, /* sine of angle of second line */
float cos2, /* cosine of angle of second line */
float p2) /* intercept representation of second line */
```

While the use of sine and cosine for each angle overspecifies the line, these numbers have already been calculated and stored, so it is more efficient to use both. It returns the x and y values of the intersection.

max_smooth

max_smooth takes a histogram of values, and returns the maximum histogram bin with the maximum value after smoothing.

```
int max_smooth (int *hist, /* The histogram array. */ int n) /* The number of histogram elements. */
```

The code implements a 1-2-1 filter twice (that is, $h_n = h_{n-1} + 2h_n + h_{n+1}$) before choosing the maximum value.

compute_scale

compute_scale computes the scale between two sets of lines.

The number returned is the scale factor in percent (100.0 is no scaling).

compute_shift

compute_shift computes the x and y translation between sets of lines.

```
/* Return val: [0] is x, [1] is y. */
float compute shift(float *retval,
                                                        /* First line endpoint list. */
                             rectlist *datal,
                                                     /* Second line endpoint list. */
                             rectlist *data2,
                             int dlength1,
                                                       /* Number of segments in first line. */
                                                       /* Number of segments in second line. */
                             int dlength2,
                             float 'length2, /* Second segment lengths. '/
float *cos1, /* First lines' cosine value of angle. */
float *cos2, /* Second lines' cosine value of angle. */
float *sin1, /* First lines' sine value of angle. */
float *sin2, /* Second lines' sine value of angle. */
float *p1, /* Contains x sin of ang - y cos of ang */
float *p2, /* Contains x sin of ang - y cos of ang */
                             float *dir1,
                                                      /* Second line's direction. */
                                                       /* Second line's direction. */
                             float *dir2,
                                                        /* Minimum line length considered. */
                             float thresh,
                             float shift_range) /* Translational difference considered. */
```

The translation values are returned in retval.

image__clear

image clear clears the elements of the image data structure for later use.

```
void image_clear (image *in) /* in is a pointer to an image data type. */
```

image__free_char

image__free_char frees the space allocated to an image with the image__allocate_char routine.

image__allocate_char

image_allocate_char allocates a 2d matrix of characters.

The function returns a pointer to the area allocated.

image__allocate_float

image_allocate_float allocates a 2d array of floating point storage.

```
float **image_allocate_float (int cc, /* Number of columns... */
int rr) /* Number of rows.... */
```

The routine returns a pointer to the allocates space.

image__write_pnm

image_write_pnm writes a pnm format graphics file.

The routine returns zero (false) if it cannot write the file, true otherwise.

image__read_pgm_header

image_read_pgm_header examines the header of a pgm (portable greymap) format graphics file.

The routine returns zero (false) if it cannot read the file header, true otherwise. It also indicates the number of columns, rows, and colors in the image through the pointers passed to it.

image__read_pgm_image

image_read_pgm_image reads the data in a pgm image.

The routine returns zero (false) if it cannot read the file, true otherwise. The actual data is returned in the image data structure.

image__alloc

image_alloc allocates the memory needed to read the image data in.

This routine modifies the image data structures.

image__read

image_read is a wrapper to perform the read. It prints out an informational message in the process.

image__free

image_free frees the allocated memory associated with the image in question.

```
void image __free (image *in) /* Pointer to the image data structure. */
```

image__follow

image__follow follows a connected edge.

This is a recursive routine, called by image_canny. It attempts to look around in an area and track adjacent points.

image__canny

image_canny performs Canny edge detection. This code is modified from the shareware developed by the Robot Vision Group, University of Edinburgh, U.K. by Bob Fisher, Dave Croft and A Fitzgibbon. The modifications were made to allow it to work in our environment.

The results are stored in the image.

image__thin

image_thin is a thinning algorithm for binary images. See 'Image Processing' by Anil K Jain (p383) for details.

```
void image__thin (image *in) /* in is the (binary) image in question. */
```

image__split

image_split is a split function for fitting poly-lines to a list

The routine is called from image_trace, and is recursive.

image__trace

image_trace is used to find lines.

It is used by image_r2v to convert to find the lines in an image.

image__r2v

image_r2v performs the raster to vector conversion.

The routine returns the number of lines identified in the image. The line data is returned in rlptr.

Sample Results

Performance Summary

The following table documents the algorithm's results on several image pairs.

Original Image	Second Image	Perceived Rotation	Perceived Scaling	Perceived Translation
ortho1.pgm	ortho1_r7.pgm	anti,7.40	99.50%	3.2,2.4
ortho1_r7.pgm	orthol.pgm	7.20	100.50%	0.0,-0.8
ortho1.pgm	ortho1_s95.pgm	0.20	94.06%	0.8,4.0
ortho1_s95.pgm	ortho1.pgm	anti,0.40	106.31%	0.0,-4.0
ortho1.pgm	ortho1_t26.pgm	anti,0.20	100.10%	26.4,-25.6
ortho1_t26.pgm	ortho1.pgm	anti,0.20	100.10%	-25.6,22.4
ortho1.pgm	ortho1_r7_s95.pgm	anti, 7.00	99.50%	-20.8,-0.8
ortho1_r7_s95.pgm	ortho1.pgm	7.00	100.50%	16.8,8.0
ortho1.pgm	ortho1_r7_t10.pgm	anti, 7.4	99,30	-4.8,12.8
ortho1_r7_t10.pgm	ortho1.pgm	7.20	102.95%	2.4,-12.8
ortho1.pgm	ortho1_s95_t10.pgm	0.20	94.44%	-11.2,12.8
ortho1_s95_t10.pgm	ortho1.pgm	anti,0.40	105.88%	11.2,-12.8
ortho1.pgm	ortho1_r7_s95_t20.pgm	anti,7.0	96.16%	-22.4,23.2
ortho1_r7_s95_t20.pgm	ortho1.pgm	6.8	102.54%	28.8,-16

Figure 1 Table of Results

Note that 'anti' rotation is counterclockwise.

Image sources: The original image is ortho1.pgm, a 1024x1024 image taken from a grayscale orthophoto obtained from the USGS. The image of an area in Carderock, Maryland, and is 2 meters per pixel. All other comparison images were generated by the noted transformations:

ortho1_r7.pgm: ortho1 rotated 7 deg, boundaries clipped and filled by pasting representative image data into the areas left uncovered by the rotation..

ortho1_s95.pgm: ortho1 scaled to 95% of original size, black (non-filled) boundaries.

ortho1 t26.pgm: ortho1 translated by (26,26), empty area filled.

ortho1_r7_s95.pgm:ortho1 rotated 7 deg, then scaled by 0.95; boundaries clipped and filled.

ortho1_r7_t10.pgm: ortho1 rotated 7 deg, then translated by (10,10); boundaries clipped and filled.

ortho1_s95_t10.pgm: ortho1 scaled by 0.95, then translated by (10,10), boundaries clipped and filled

ortho1_r7_s95_t20.pgm: ortho1 rotated 7 deg, then scaled by 0.95, then translated by (20,20); boundaries clipped and filled.

Note that most of the results are reasonable. One of the exceptions is the case where the comparison image was ortho1_r7_s95.pgm. In this case, the rotation was found accurately, but the scaling was not. Curiously, when this image was then translated as in ortho1_r7_s95_t20.pgm, the perceived scaling values were much

better. Examination of the histograms of relative ratio of areas between the triangles used for scale determination indicates that in this set of images, the 'correct' scale is not greatly higher than the nearby scales. Additionally, in the images ortho1_r7_s95_t20.pgm and ortho1_r7_s95.pgm the boundaries created by the transformation were filled in by hand separately in each case. Several lines were found in the hand-drawn areas in the non-translated image that were not present in the translated image, and these contributed to the ratio of areas enough to skew the results to a different histogram bin.

Detailed Summary

In this section, the intermediate results from a single pair of images are presented.

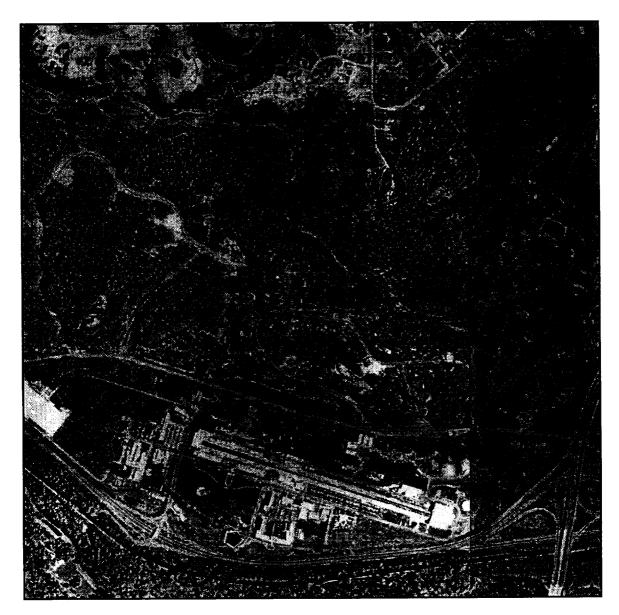


Figure 2 First source image: ortho1.pgm

The first image, orthol.pgm, is taken from a larger USGS orthophoto.

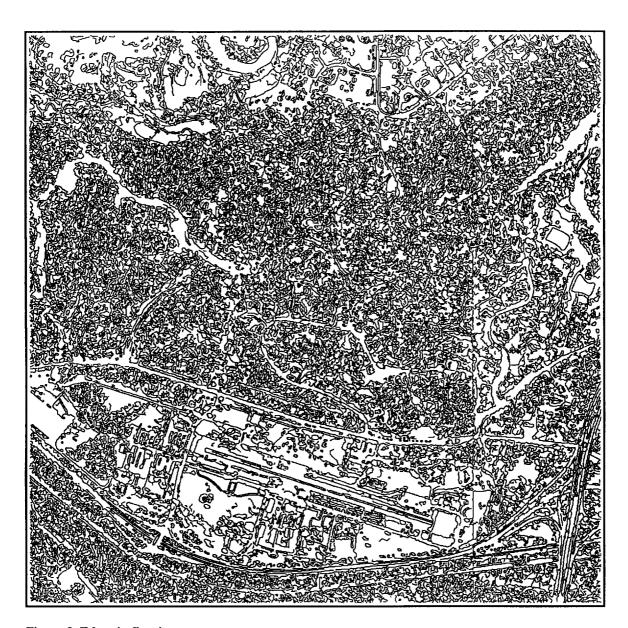


Figure 3 Edges in first image

The above figure shows the edges detected by the Canny operation on the first source image.



Figure 4 Lines detected in ortho1.pgm

The lines detected in the first source image are shown above, overlaid on the original. The lines that are below threshold are colored red, and the lines used are in green.

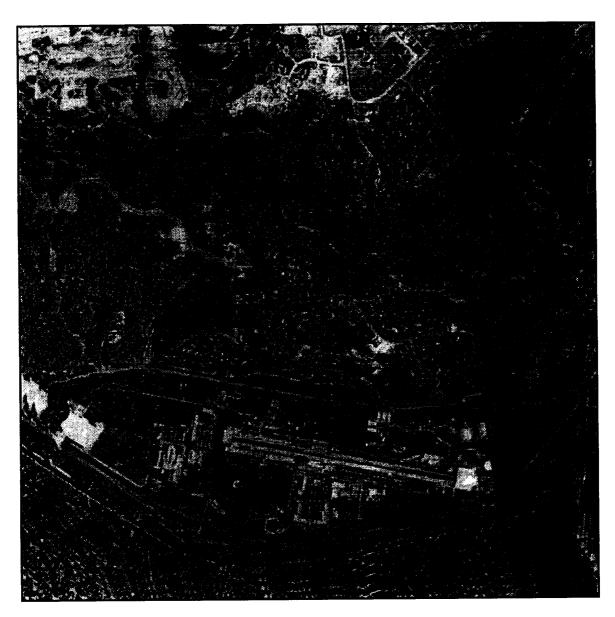


Figure 5 Second source image: ortho1_r7_s95_t20.pgm

The above image is the second source used for this test. It is a rotated, translated, and scaled version of the original, with cloned trees filling in the areas around the edges created by shrinking and scaling the original.

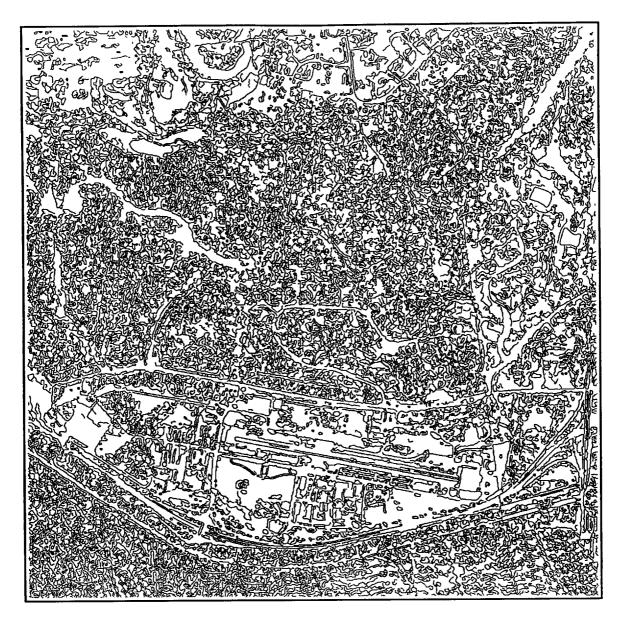


Figure 6 Edges detected in second image

The above figure shows the edges detected by the Canny operation on the first source image.



Figure 7 Lines detected in second image

The lines detected in the second source image are shown above, overlaid on the original. The lines that are below threshold are colored red, and the lines used are in blue.

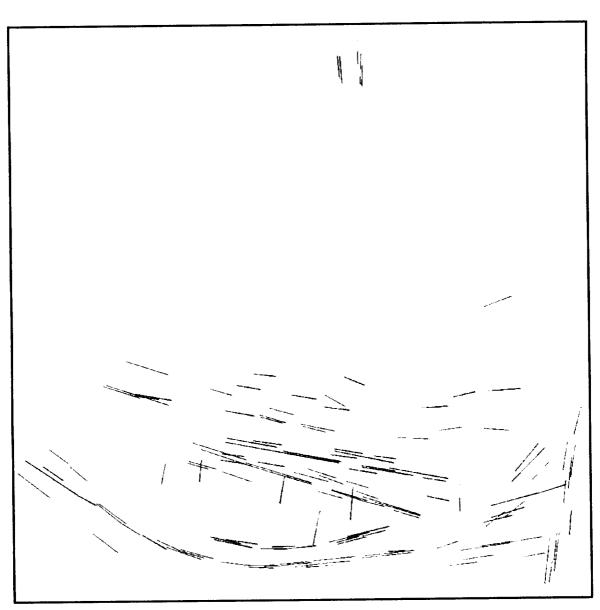


Figure 8 Lines from both images

The lines from both images are shown together in the above image (green for the first image, blue for the second). Only line lengths above the threshold are shown.

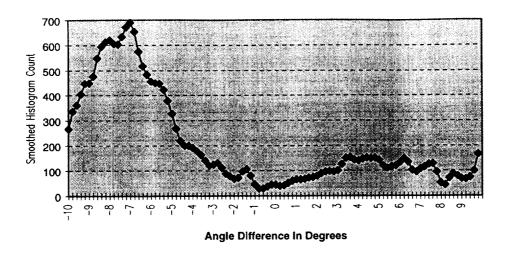


Figure 9 Angle Difference Histogram

The above figure shows the smoothed histogram of angle difference values. Based on the peak value in the bin at -7, this value was chosen to be the rotational angle between images.

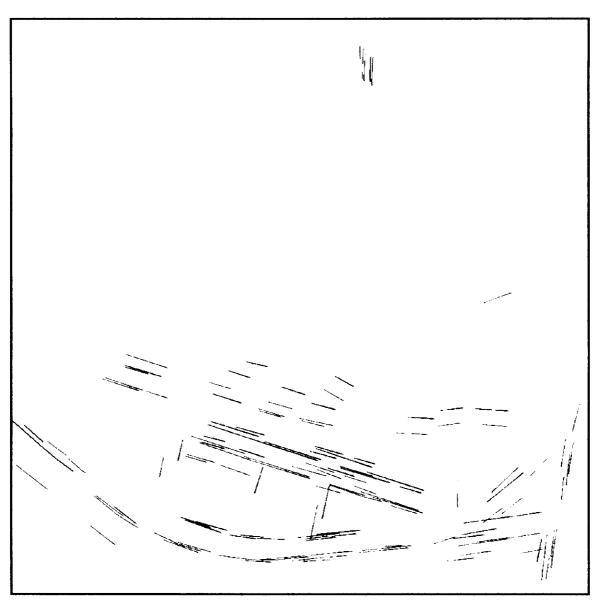


Figure 10 Lines from both images after derotation

The lines from both images are shown again, after the lines in the second image (blue) are rotated to remove the rotational difference between the two images.

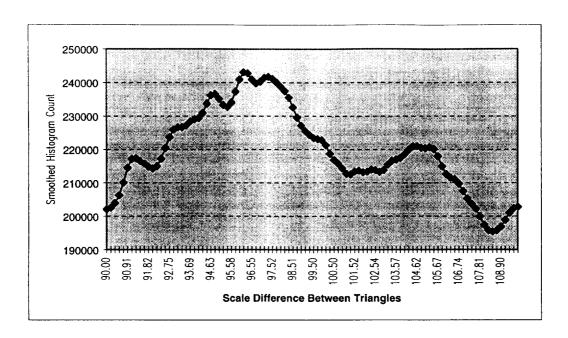


Figure 11 Scale Difference Histogram

The above figure shows the smoothed histogram of relative scales of the triangles. Based on the peak value in the bin at 96.16%, this value was chosen to be the scale factor between images.

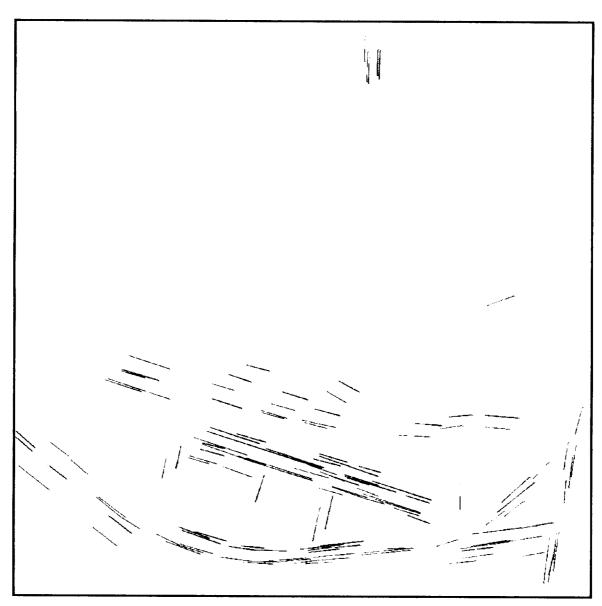


Figure 12 Lines from both images after descaling

The line pairs are shown above after compensation for the second image's scaling factor.

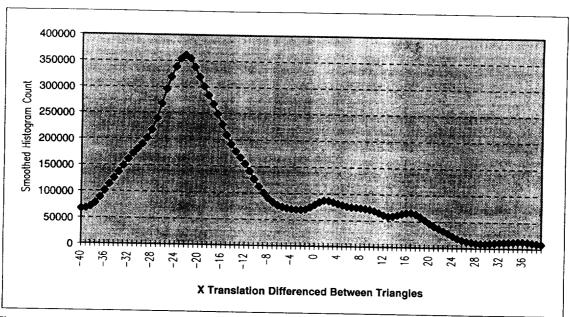


Figure 13 Translation Values in X

The above figure shows the smoothed histogram of the translation values in the X axis between the set of triangles. Based on the peak at -22.4, this value was chosen to be the translation on the X axis.

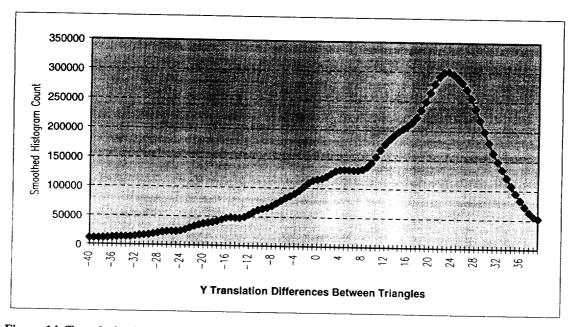


Figure 14 Translation Values In Y

The above figure shows the smoothed histogram of the translation values in the Y axis between the set of triangles. Based on the peak at 23.2, this value was chosen to be the translation on the Y axis.

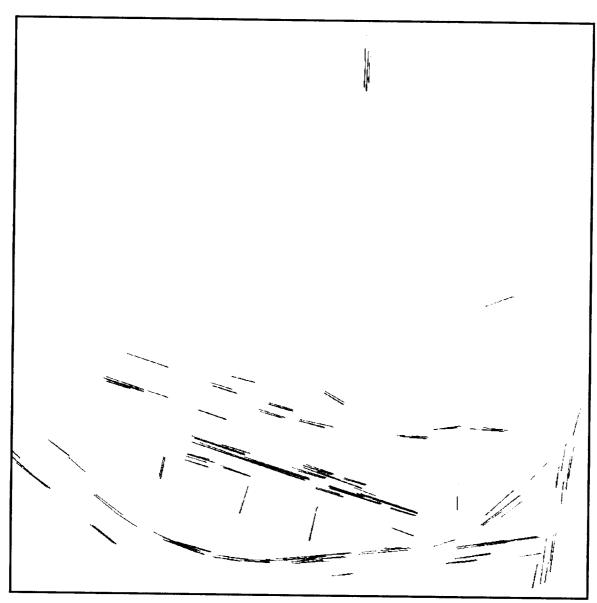


Figure 15 Lines from both images after detranslation

The above image shows the lines after removing the translation factors determined by the program.

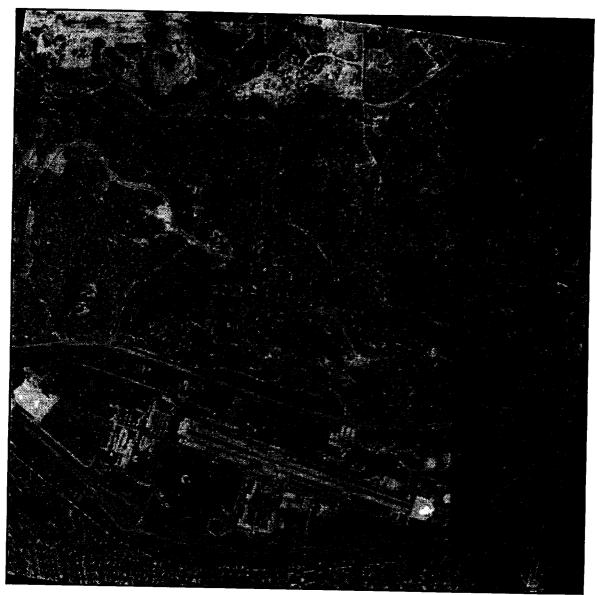


Figure 16 Second image transformed to correspond to first

The above image shows the final result of the process: the second image transformed such that it corresponds to the first image.

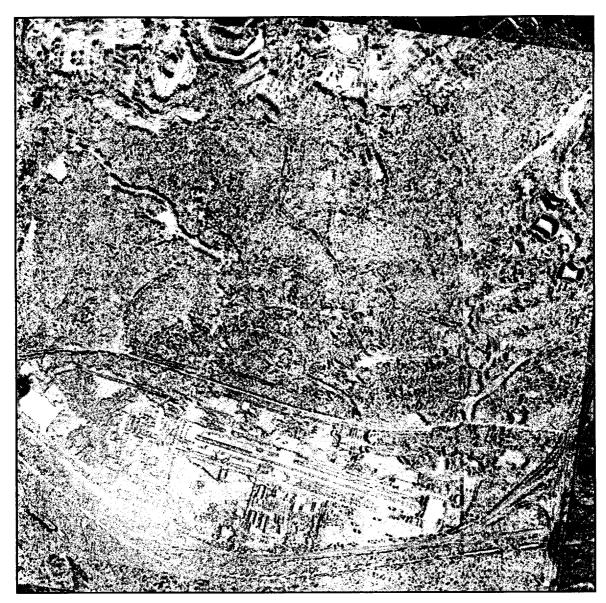


Figure 17 Difference image of result and ortho1.pgm

The difference image between the original and output images is shown above (where white is no difference, black is maximal difference). The areas of maximum difference are the regions in the output image that mapped to areas outside the original second image, and so were colored black.